GLASS FUNDAMENTALS 3A
UNDERSTANDING EMISSIVITY

Radiative heat transfer will occur from an object, as its relative temperature to surrounding objects, and the atmosphere, becomes greater. Radiative heat transfer involves the transfer of energy without physical contact, or transfer through a medium such as a gas or liquid.

When considering radiative heat transfer, the emissivity of a material or surface is a key parameter. This Technical Document outlines the fundamentals associated with emissivity with regards to radiative heat transfer, as well as provides the derivation for radiative heat transfer between two parallel surfaces.

EMISSIVITY

Emissivity is defined as the ratio of energy radiated from the surface of a material relative to a perfect emitter (black body), at the same temperature and wavelength, and under the same viewing conditions. As such, if we consider the Stefan-Boltzmann Law [1, 2] where for a black body, radiative power (q_{bb});

$$q_{bb} = \sigma \cdot T^4$$

Where; σ Stefan-Boltzmann Constant, 5.6703 x 10^{-8} W/m^2.K^4

T Surface Temperature (K)

Then for a non-perfect emitter, or grey body, radiative power (q_{gb});

$$q_{gb} = \varepsilon \cdot \sigma \cdot T^4$$

Where; ε Surface Emissivity

MEASUREMENT OF EMISSIVITY

The normal emissivity for coated glass products is determined in accordance with EN 12898 [3], and typically involves the measurement of the infrared normal reflectance of a sample, between 5 – 50 µm against a reference standard of known properties, typically electroplated gold. Assuming no infrared transmittance, the normal emissivity of the samples will be determined by;

$$\varepsilon = 1 - \rho$$

NORMAL VS. HEMISPHERICAL EMISSIVITY

Normal emissivity refers to the emissivity determined from reflectance at a near normal angle of incidence, and as such is limited to a single direction for emittance. Hemispherical, or corrected, emissivity is relevant to the total emittance, in that it considers emittance in all directions, as illustrated 2-dimensionally below;

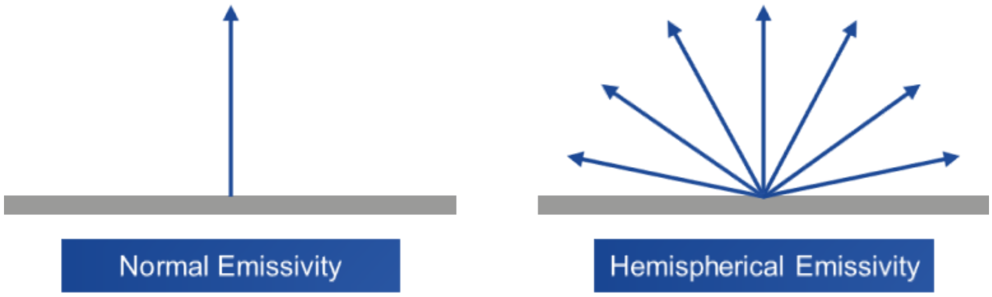

Figure 1 - 2-D Representation of Normal and Hemispherical Emissivity

A table is provided within EN 12898 for correcting normal emissivity (ε_n) to hemispherical emissivity (ε_c). This can be fitted against a 2nd order polynomial, using (0,0) as an intercept to give the following;

$$\varepsilon_c = 1.1887 \cdot \varepsilon_n - 0.4967 \cdot \varepsilon_n^2 + 0.2452 \cdot \varepsilon_n^3$$

Work by Hartmann et al. [4] also provides a correlation, which provides a close match to that from EN 12898, as below;

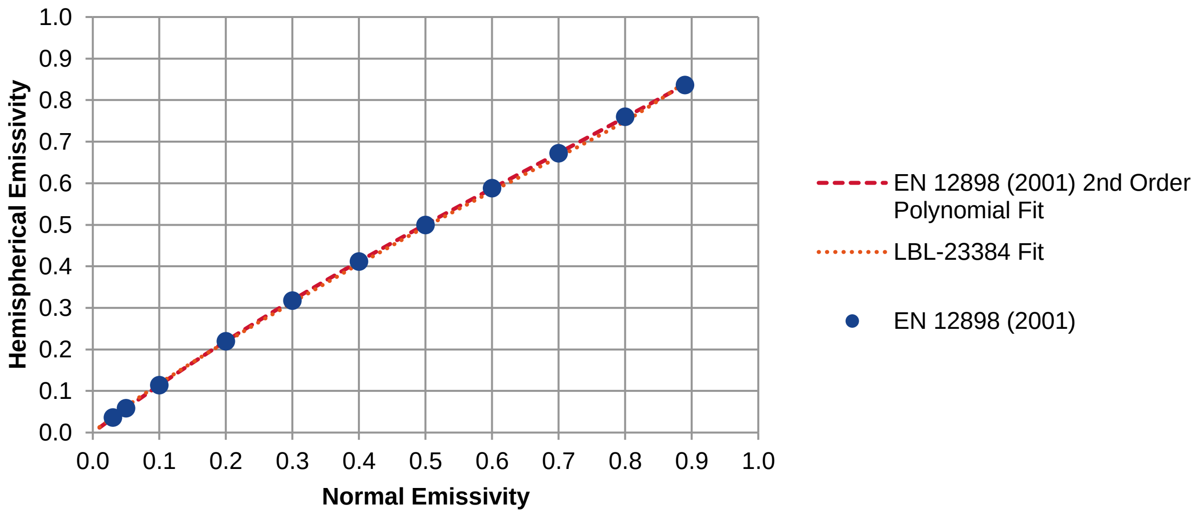

Figure 2 - Normal and Hemispherical Emissivity Relationship

DERIVATION OF RADIATIVE HEAT TRANSFER BETWEEN TWO PARALLEL SURFACES

If two planar surfaces are considered, both grey bodies, as would be the condition within an insulating glazed unit, consideration has to be given to the relative surface temperatures and emissivities. The following relationship applies;

$$q_{1\rightarrow 2} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

In order to understand the basis of the radiant heat transfer equations, consideration can be given to two grey bodies bounding a space, as illustrated below, and the behaviour of thermal energy emitted by one surface.

Assuming both surfaces are not transparent to thermal energy, then thermal energy emitted from surface 1 will be partly absorbed, and partly reflected by surface 2. The reflected thermal energy will then be partly absorbed, and partly reflected by surface 1. And again, the reflected thermal energy from surface 1 will again be incident on surface 2. This process is illustrated below.

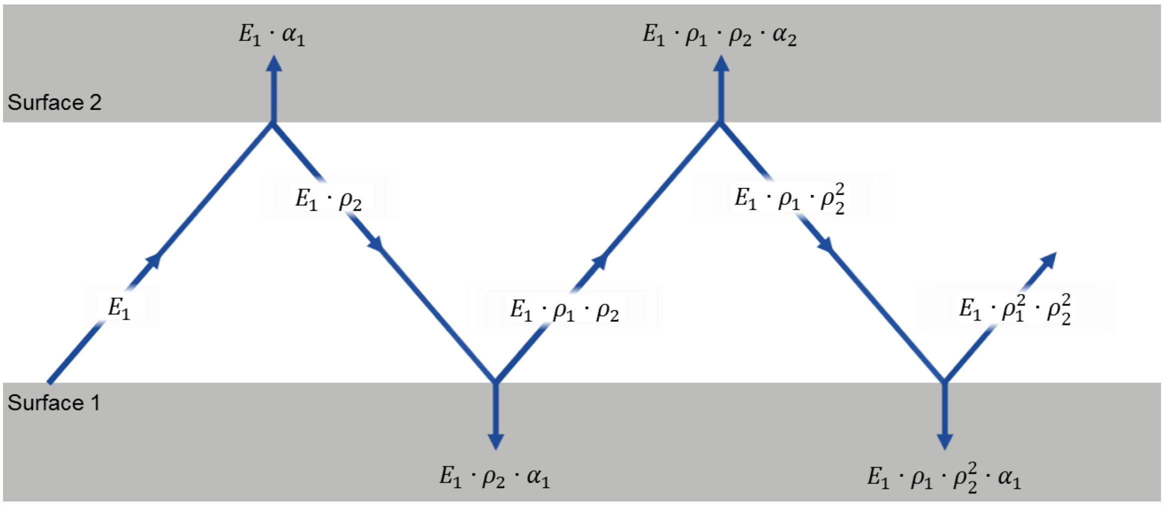

Figure 3 - Radiative Energy Transfer between Parallel Surfaces

The same sequence is true for thermal energy being emitted from surface 2, and both can be considered by the following equations;

Surface 1 Emits:	E_1	Surface 2 Emits:	E_2
Surface 2 Absorbs:	$E_1 \cdot \alpha_2$	Surface 1 Absorbs:	$E_2 \cdot \alpha_1$
Surface 2 Reflects:	$E_1 \cdot \rho_2$	Surface 1 Reflects:	$E_2 \cdot \rho_1$
Surface 1 Absorbs:	$E_1 \cdot \rho_2 \cdot \alpha_1$	Surface 2 Absorbs:	$E_2 \cdot \rho_1 \cdot \alpha_2$
Surface 1 Reflects:	$E_1 \cdot \rho_2 \cdot \rho_1$	Surface 2 Reflects:	$E_2 \cdot \rho_1 \cdot \rho_2$
Surface 2 Absorbs:	$E_1 \cdot \rho_2 \cdot \rho_1 \cdot \alpha_2$	Surface 1 Absorbs:	$E_2 \cdot \rho_1 \cdot \rho_2 \cdot \alpha_1$
Surface 2 Reflects:	$E_1 \cdot \rho_2^2 \cdot \rho_1$	Surface 1 Reflects:	$E_2 \cdot \rho_1^2 \cdot \rho_2$
Surface 1 Absorbs:	$E_1 \cdot \rho_2^2 \cdot \rho_1 \cdot \alpha_1$	Surface 2 Absorbs:	$E_2 \cdot \rho_1^2 \cdot \rho_2 \cdot \alpha_2$
Surface 1 Reflects:	$E_1 \cdot \rho_2^2 \cdot \rho_1^2$	Surface 2 Reflects:	$E_2 \cdot \rho_1^2 \cdot \rho_2^2$
…continues			

To simplify the equations, consideration can be given to the lack of transparency to thermal radiation, and as such if;

$$\alpha + \rho = 1$$

Therefore for thermal energy emitted from surface 1;

Surface 1 Emits:	E_1
Surface 2 Absorbs:	$E_1 \cdot \alpha_2$
Surface 2 Reflects:	$E_1 \cdot (1-\alpha_2)$
Surface 1 Absorbs:	$E_1 \cdot (1-\alpha_2) \cdot \alpha_1$
Surface 1 Reflects:	$E_1 \cdot (1-\alpha_2) \cdot (1-\alpha_1)$
Surface 2 Absorbs:	$E_1 \cdot (1-\alpha_2) \cdot (1-\alpha_1) \cdot \alpha_2$
Surface 2 Reflects:	$E_1 \cdot (1-\alpha_2) \cdot (1-\alpha_2) \cdot (1-\alpha_1)$
Surface 1 Absorbs:	$E_1 \cdot (1-\alpha_2) \cdot (1-\alpha_2) \cdot (1-\alpha_1) \cdot \alpha_1$
Surface 1 Reflects:	$E_1 \cdot (1-\alpha_2) \cdot (1-\alpha_2) \cdot (1-\alpha_1) \cdot (1-\alpha_1)$
…continues	

The same can be applied to thermal energy emission from surface 2. To simplify the equations the following relationship can be used;

$$\beta = (1-\alpha_1)(1-\alpha_2)$$

Therefore, the absorption on surface 1 can be defined as the sum of;

Surface 1 Absorbs:	$E_1 \cdot (1-\alpha_2) \cdot \alpha_1$
Surface 1 Absorbs:	$E_1 \cdot (1-\alpha_2) \cdot \beta \cdot \alpha_1$
Surface 1 Absorbs:	$E_1 \cdot (1-\alpha_2) \cdot \beta^2 \cdot \alpha_1$
…continues	

Leading to;

$$\text{Surface 1 Emitted \& Absorbed} = E_1 \cdot (1-\alpha_2) \cdot \alpha_1 \cdot [1 + \beta + \beta^2 + \beta^3 + \cdots]$$

To further simply this, the following relationship can be defined;

$$\frac{1}{1-\beta} = 1 + \beta + \beta^2 + \beta^3 + \cdots$$

Leading to;

$$\text{Surface 1 Emitted \& Absorbed} = \frac{E_1 \cdot (1-\alpha_2) \cdot \alpha_1}{1-\beta}$$

And for surface 2;

$$\text{Surface 2 Emitted \& Absorbed} = \frac{E_2 \cdot (1-\alpha_1) \cdot \alpha_2}{1-\beta}$$

Resulting in;

$$\text{Surface 1 Absorbed} = E_2 - \frac{E_2 \cdot (1-\alpha_1) \cdot \alpha_2}{1-\beta} = \frac{E_2 \cdot \alpha_1}{1-\beta}$$

Therefore, the flux from surface 1 to surface 2 can be defined by;

$$\dot{q}_{1\to2} = E_1 - \frac{E_1 \cdot (1-\alpha_2) \cdot \alpha_1}{1-\beta} - \frac{E_2 \cdot \alpha_1}{1-\beta}$$

As;

$$\beta = (1-\alpha_1)(1-\alpha_2)$$

Then;

$$\dot{q}_{1\to2} = \frac{E_1 \cdot \alpha_2 - E_2 \cdot \alpha_1}{\alpha_1 + \alpha_2 - \alpha_1 \cdot \alpha_2}$$

With consideration the relationship between a grey body and emissivity, as per Stefan-Boltzmann's Law [1, 2], where;

$$E = \varepsilon \cdot \sigma \cdot T^4$$

And Kirchoff's Law of thermal radiation [3], where, for a grey body;

$$\varepsilon = \alpha$$

Therefore;

$$\dot{q}_{1\to2} = \frac{\varepsilon_1 \cdot \sigma \cdot T^4 \cdot \varepsilon_2 - \varepsilon_2 \cdot \sigma \cdot T^4 \cdot \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \cdot \varepsilon_2}$$

Which simplifies to;

$$\dot{q}_{1\to2} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

REFERENCES

[1] J. Stefan, “Über die Beziehung zwischen der Wärmestrahlung und der Temperatur,” *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften in Wienn,* vol. 79, pp. 391-428, 1879.

[2] L. Boltzmann, “Ableitung des Stefan'schen Gesetzes, betreffend die Abhängigkeit der Wärmestrahlung von der Temperatur aus der electromagnetischen Lichttheorie,” *Annalen der Physik,* vol. 258, no. 6, pp. 291-294, 1884.

[3] G. Kirchoff, “Ueber das Verhältniss zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme und Licht,” *Annalen der Physik,* vol. 185, no. 2, pp. 275-301, 1860.

[4] European Committee for Standardization, EN 12898:2001 - Glass in building. Determination of the emissivity, CEN, 2001.