GLASS FUNDAMENTALS 2A LINEAR VS. NON-LINEAR ANALYSIS OF GLASS PLATES

A plate is typically defined as a structure with two plane faces, separated by a thickness, where the thickness is small relative to the other dimensions, specifically the width and height in the case of a rectangular plate. There are numerous methodologies that can be used to determine the deflection, and associated stress, resultant from a plate under loading, under varying support conditions.

This document will give a brief overview of the differences observed when assessing plates through linear and non-linear methods, as well as some of the theory behind this.

LINEAR LIMITS FOR GLASS DEFLECTION

Linear plate theory was established by Kirchhoff in 1850 [1], and is considered suitable where the deflections relative to the plate thickness are small. Where the deflection exceeds the plate thickness, linear plate theory loses accuracy, and a geometrically non-linear plate theory approach, as first derived by Von Kármán [2], becomes necessary.

For flexible plates, such as glass, whether monolithic or laminated, as the deflection begins to exceed a certain level relative to the thickness of the plate, membrane forces will begin to influence the behaviour, and this is where the non-linear behaviour begins.

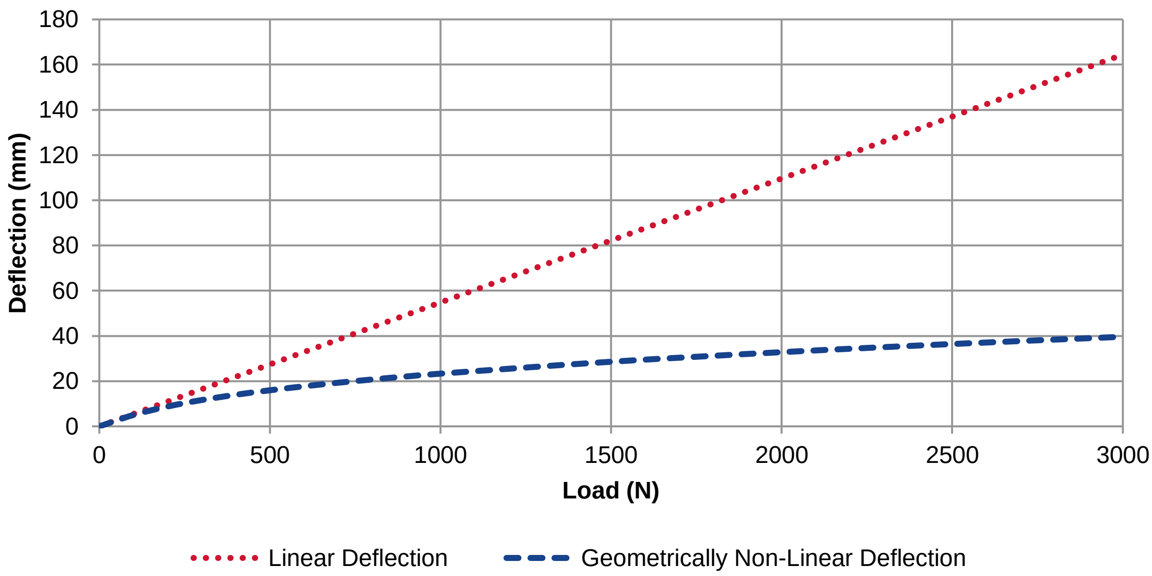

Figure 1 - Plate Deflection Determined by Linear & Non-Linear Methods (1)

If we consider the deflection of a glass plate, 1200 mm x 2500 mm, and 4 mm thick, four edge simply supported, and under a uniform load, Figure 1 shows the deflection calculated from both linear and geometrically non-linear plate theories within finite element analysis.

As can be observed, as the load, and so deflection, increases, the linear analysis results in a significantly higher deflection that for the geometrically non-linear approach. Figure 2 shows the lower deflection region of the previous example, where the divergence can be seen to begin increasing more significantly from 2 mm, which is half the plate thickness.

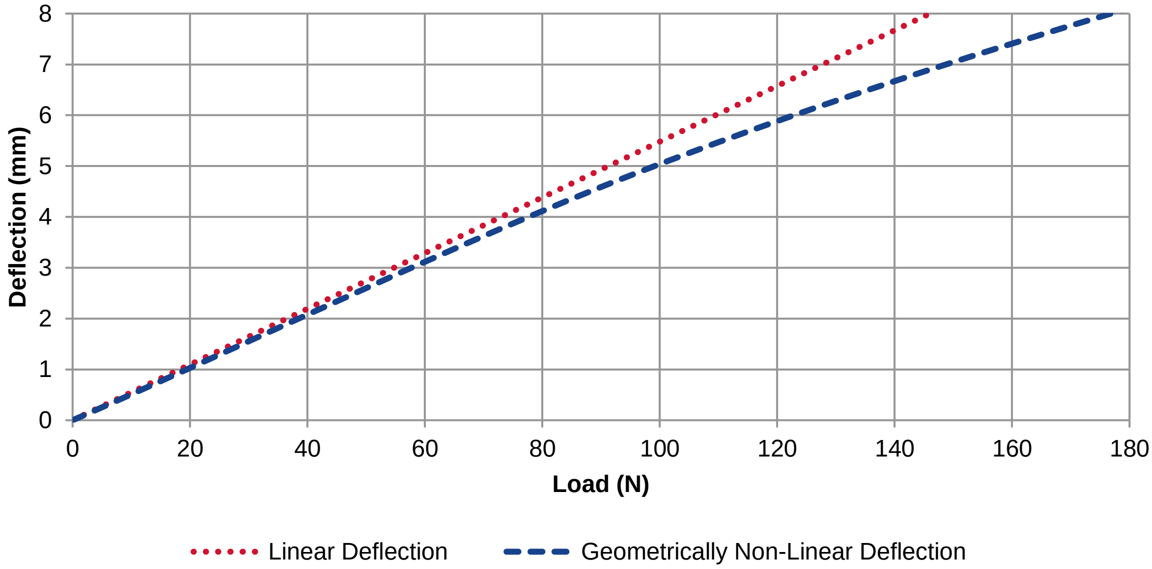

Figure 2 - Plate Deflection Determined by Linear & Non-Linear Methods (2)

Within prEN 16612 [3], this limit is stated to be as follows; “Where the deflection induced by the actions exceeds half the glass thickness, linear theory of plate bending may excessively overestimate the stresses and the maximum deflection. In this case the stress distribution and the maximum deflection can be calculated according to non-linear plate theory.”

INFLUENCE ON STRESS & DEFLECTION

With geometrically non-linear theory, the membrane forces are considered within the glass, as well as the associated increase in the stiffness of the plate due to these forces. Membrane forces will influence both the deflection, and the resultant stresses calculated.

DEFLECTION

As observed in Figure 1 and Figure 2, the deflection is reduced when considering a geometrically non-linear approach for uniform loads. The same effect can be observed for other load types, such as line loads and concentrated loads, and Figure 3 shows the same plate properties as below, with an applied concentrated load.

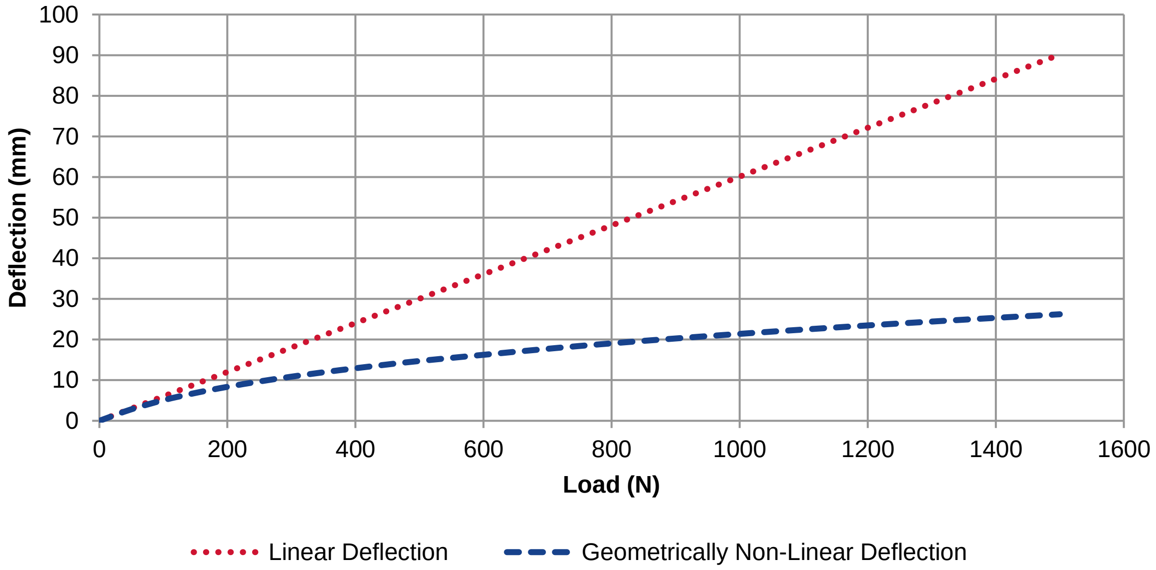

Figure 3 - Plate Deflection Determined by Linear & Non-Linear Methods (3)

STRESS

For most glass and glazing assessments, where loadings are considered it's the resultant tensile stresses that are considered with regards fracture and failure. The following shows the peak stress generated for the initial example, of a plate under uniform loading;

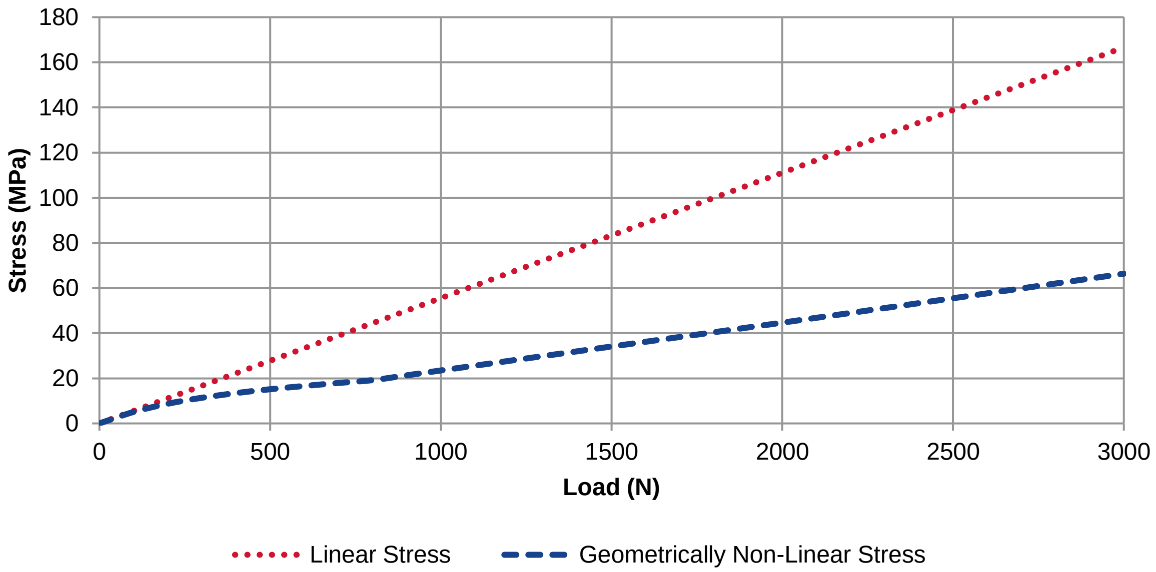

Figure 4 - Plate Tensile Stress Determined by Linear & Non-Linear Methods (1)

The surface stress distribution should also be considered, as the peak stresses, with consideration to the internal stresses in the glass, may not occur at the centre, where deflection is greatest. The below shows the stress distribution of the surface opposing the load, so expected to be under tension, for both linear and non-linear analysis;

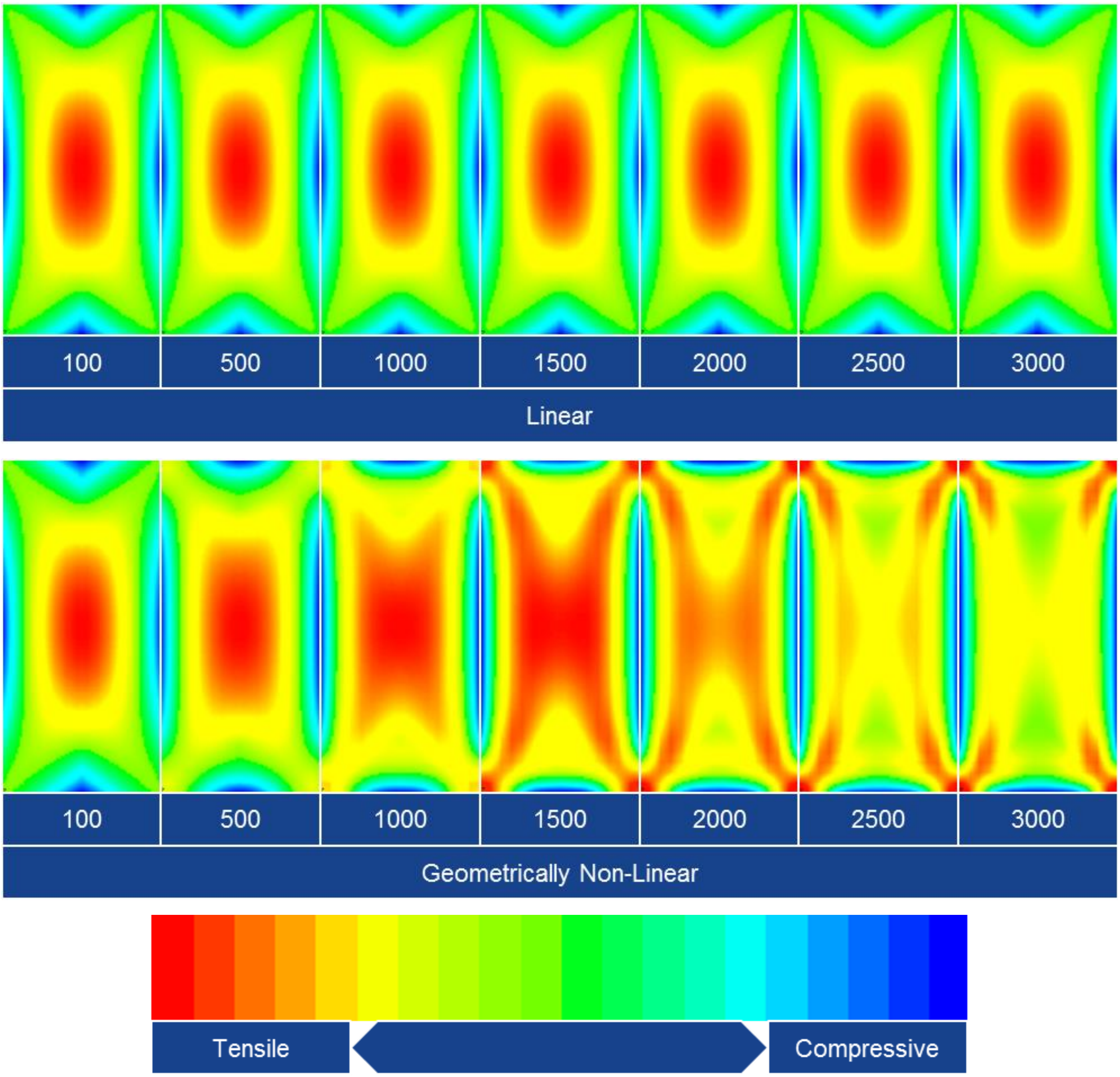

Figure 5 - Surface Stress Distributions Under Uniform Load

As can be seen, the stress distribution remains unchanged under linear analysis, demonstrating that there is no determination of the internal stresses, and how they influence the surface stresses.

APPLICATION TO GLASS & GLAZING

For the majority of glass and glazing assessments, there will be a potential for deflection levels greater than the glass thickness, and as a result of this, non-linear methods would be expected to be used.

There would be very few cases where linear methods may be considered applicable, and regardless, as a non-linear approach is also accurate at lower deflections, this approach is valid for all scenarios.

REFERENCES

[1] G. Kirchhoff, “Über das Gleichgewicht und die Bewegung einer elastischen Scheibe,” *Journal für die reine und angewandte Mathematik,* vol. 40, pp. 51-88, 1850.

[2] T. Von Kármán, “Festigkeitsprobleme im maschinenbau,” *Encycl de Math Wiss,* vol. 4, pp. 348-351, 1910.

[3] European Committee for Standardization, prEN 16612:2013 - Glass in Building - Determination of the load resistance of glass panes by calculation and testing, CEN, 2013.

[4] E. Wölfel, “Nachgiebiger Verbund: eine Näherungslösung und deren Anwendungsmöglichkeiten,” *Stahlbau,* vol. 56, no. 6, pp. 173-180, 1987.

[5] L. Galuppi and G. Royer-Carfagani, “The Effective Thickness of Laminated Glass Plates,” *Mechanics of Materials and Structures,* vol. 7, no. 4, pp. 375-399, 2012.

[6] E. Ventsel and T. Krauthammer, Thin Plates and Shells: Theory, Analysis and Applications, CRC Press, 2001.